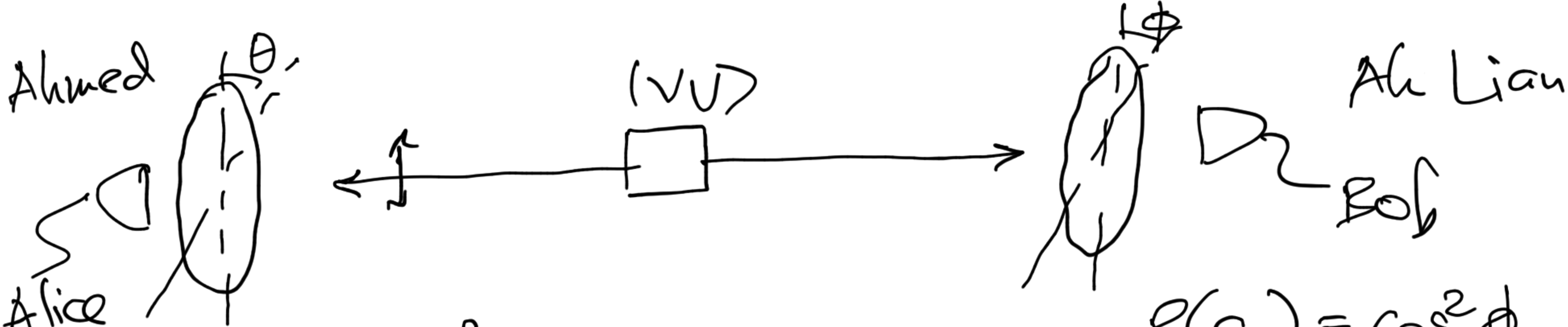
Equals: Quantum Entanglement Etc Nov 3

$\hat{a}|\alpha\rangle \rightarrow |\alpha\rangle$

$\left.\begin{array}{ll} \hat{a}+\hat{a}^{\dagger} : & \text{amplitude quadrature} \\ \hat{a}-\hat{a}^{\dagger} & \text{phase quadrative} \end{array}\right\}$ CV quantum information

$$V = \frac{\langle a_i a_j + a_j a_i \rangle}{2} - \langle a_i \rangle \langle a_j \rangle \quad \{a_i, a_i^{\dagger}\}$$

$$\underbrace{V + \frac{i\Omega}{2} > 0} \qquad \Omega \equiv \begin{bmatrix} \Omega_1 & & \\ & \Omega_2 & \\ & & \ddots \end{bmatrix} \quad \Omega_j = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



$p(\text{clicks}) = \cos^2\theta$

$p(nc) = \sin^2\theta$

$P(c) = \cos^2\phi$

$P(nc) = \sin^2\phi$

		Corr Value
$P(\underbrace{\text{clicks}, \text{clicks}})$	$= \cos^2\theta\cos^2\phi$	1
$P(c, nc)$	$= \cos^2\theta\sin^2\phi$	-1
$P(nc, c)$	$= \sin^2\theta\cos^2\phi$	-1
$P(nc, nc)$	$= \sin^2\theta\sin^2\phi$	1

$$\left.\begin{array}{ccr} c & c & 1 \\ c & nc & -1 \\ c & nc & -1 \\ nc & nc & 1 \\ nc & c & -1 \\ \vdots & & \vdots \end{array}\right\}$$

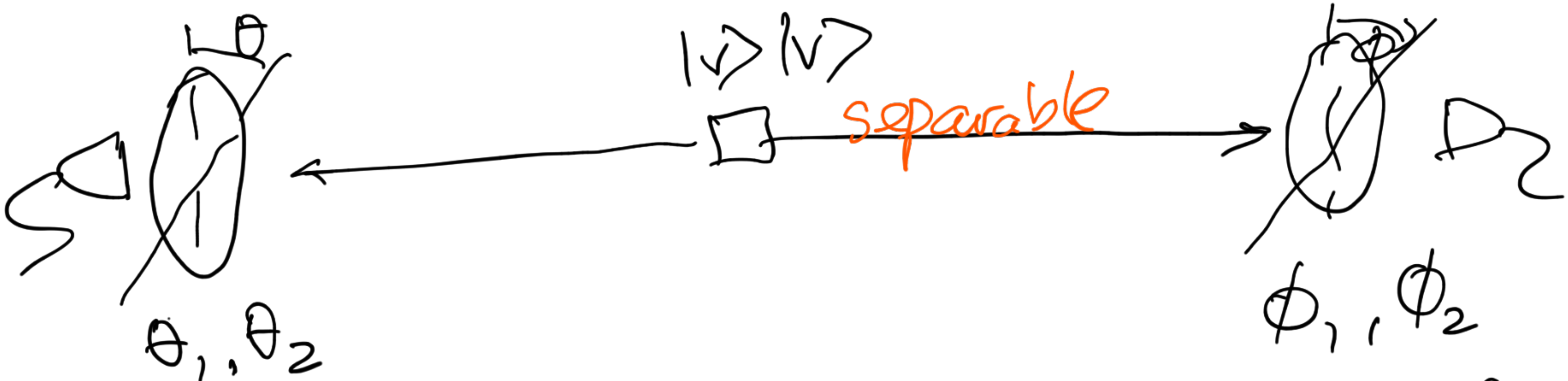
Average

$$\left.\begin{array}{lr} \cos^2\theta \cos^2\phi & 1 \\ \cos^2\theta \sin^2\phi & -1 \end{array}\right\}$$

$$\left.\begin{array}{lr} = \sin^2\theta \cos^2\phi & -1 \\ = \sin^2\theta \sin^2\phi & 1 \end{array}\right\}$$

$$\langle E \rangle = \cos^2\theta \underbrace{\left(\cos^2\phi - \sin^2\phi\right)} + \sin^2\theta \underbrace{\left(\sin^2\phi - \cos^2\phi\right)}$$

Average Correlation $= \cos 2\theta \cos 2\phi$



$$\left| \underbrace{E(\theta_1,\phi_1)}_{-1 \le \quad \le 1} + E(\theta_1,\phi_2) + E(\theta_2,\phi_1) + E(\theta_2,\phi_2) \right| \le 4$$

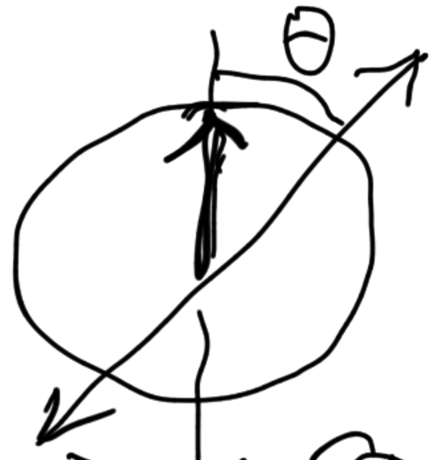
$$B = \left| E(\theta_1,\phi_1) + E(\theta_1,\phi_2) + \underbrace{E(\theta_2,\phi_1) - E(\theta_2,\phi_2)}_{\le 2} \right|$$

$E(\theta_1,\phi_1) \to \cos 2\theta_1 \cos 2\phi_1$

$E(\theta_1,\phi_2) \to \cos 2\theta_1 \cos 2\phi_2$

$$0 = \frac{\partial B}{\partial \theta_1} = \frac{\partial B}{\partial \theta_2} = \frac{\partial B}{\partial \phi_1} = \frac{\partial B}{\partial \phi_2}$$

$|v\rangle_A |v\rangle_B$

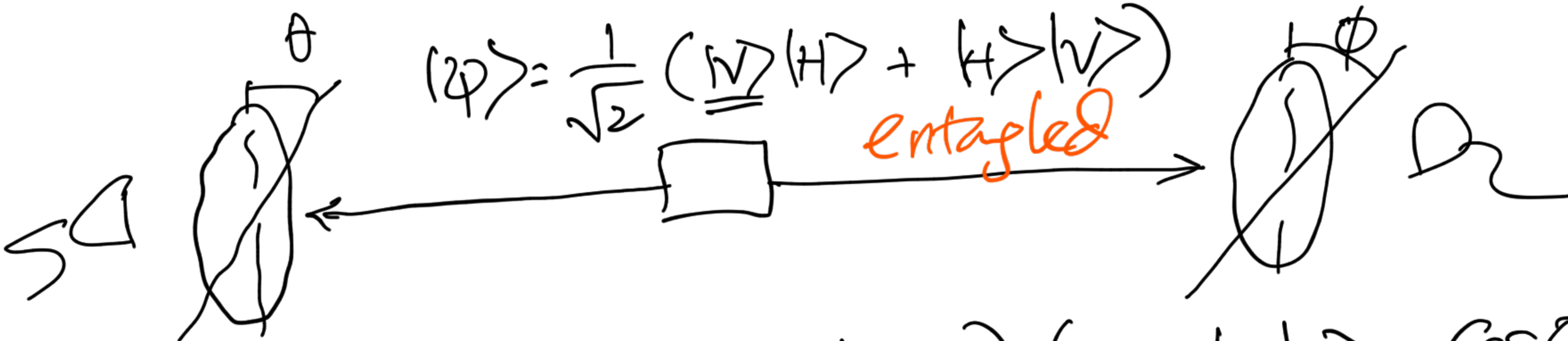


$$(\cos\theta\,|c\rangle + \sin\theta\,|nc\rangle)(\cos\phi\,|c\rangle + \sin\phi\,|nc\rangle)$$

$$= \underbrace{\cos\theta\cos\phi}_{}\,|cc\rangle + \cos\theta\sin\phi\,|c,nc\rangle + \sin\theta\cos\phi\,|nc,c\rangle + \sin\theta\sin\phi\,|nc,nc\rangle$$

$$P(c,c) = \cos^2\theta\cos^2\phi$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|V\rangle|H\rangle + |H\rangle|V\rangle\right)$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\left(\cos\theta\,|c\rangle + \sin\theta\,|nc\rangle\right)\left(\sin\phi\,|c\rangle - \cos\phi\,|nc\rangle\right) + \left(\sin\theta\,|c\rangle - \cos\theta\,|nc\rangle\right)\left(\cos\phi\,|c\rangle + \sin\phi\,|nc\rangle\right)\Big)$$

$$= \frac{1}{\sqrt{2}}\Big(\sin(\theta+\phi)\,|c,c\rangle - \cos(\theta+\phi)\,|c,nc\rangle - \cos(\theta+\phi)\,|nc,c\rangle - \sin(\theta+\phi)\,|nc,nc\rangle\Big)$$

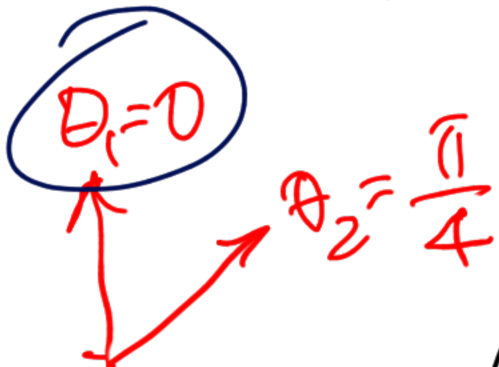
$(1)\; P(c,c) = \frac{1}{2}\sin^2(\theta+\phi)$ $\quad (-1)\; P(nc,c) = \frac{1}{2}\cos^2(\theta+\phi)$

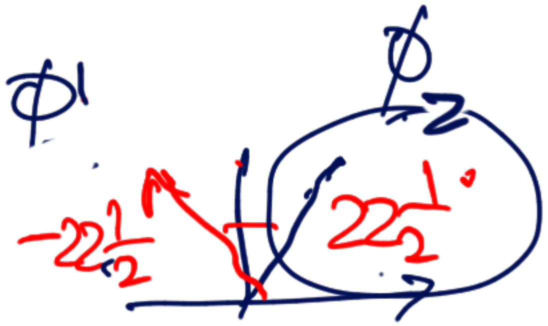
$(-1)\; P(c,nc) = \frac{1}{2}\cos^2(\theta+\phi)$ $\quad (+1)\; P(nc,nc) = \frac{1}{2}\sin^2(\theta+\phi)$

$$E(\theta,\phi) = -\cos 2(\theta+\phi)$$

$$f(\theta)\,g(\phi)$$

$$\left|E(\theta_1,\phi_1) + E(\theta_2,\phi_1) + E(\theta_1,\phi_2) + E(\theta_2,\phi_2)\right| \le 4$$





$\cos 2(\theta_2+\phi_1)$

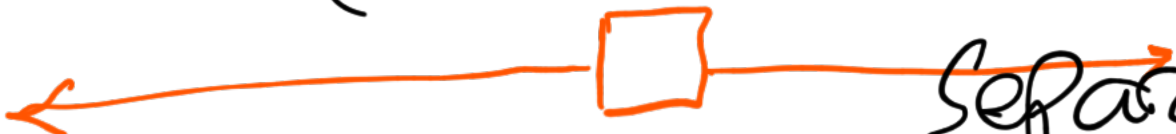
$$B = \left|E(\theta_1,\phi_1) + E(\theta_2,\phi_1) + E(\theta_1,\phi_2) - E(\theta_2,\phi_2)\right|$$

$\cos 2(\theta_1+\phi_1)$ $\quad \frac{1}{\sqrt{2}}$ $\quad \cos 2(\theta_1+\phi_2)$ $\quad -\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}}$ $\quad \frac{1}{\sqrt{2}}$

$$\le \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$(\cos\alpha|0\rangle+\sin\alpha|1\rangle)(\cos\beta|0\rangle+\sin\beta|1\rangle)$$

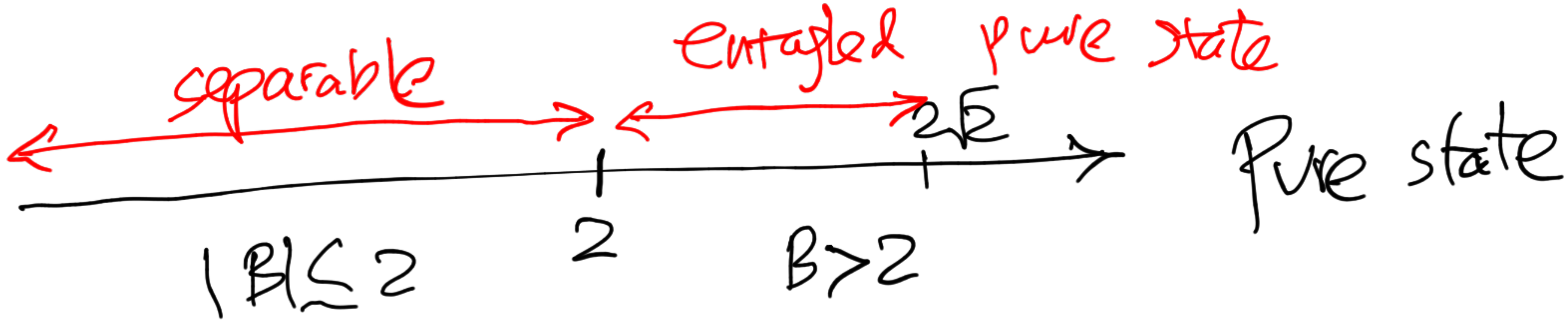


separable

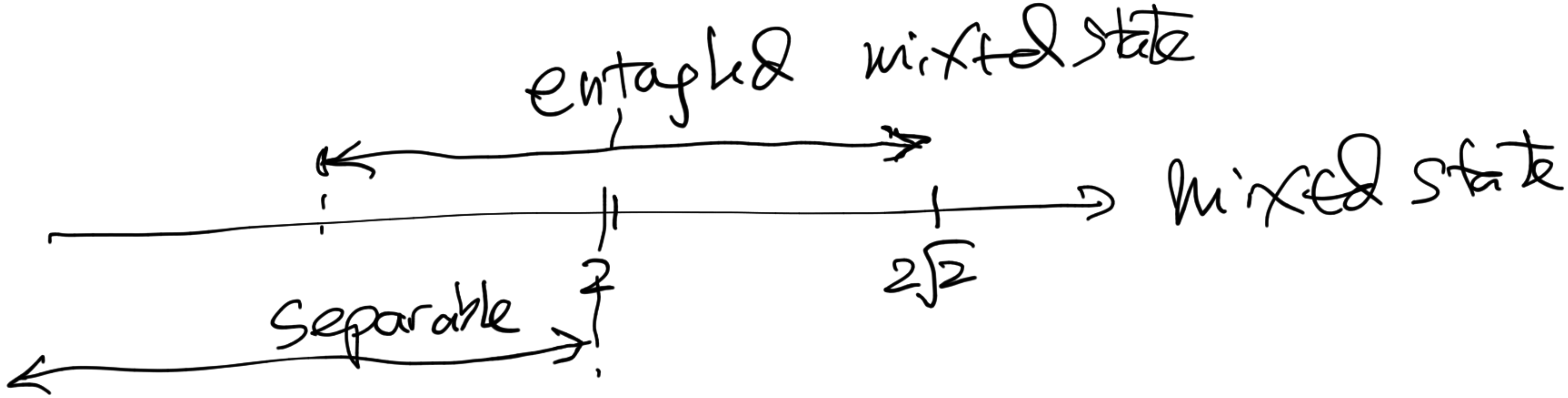
$|B| \leq 2$

entangled $|B| > 2$

$|B| \leq 2\sqrt{2}$



Bell Inequality (CHSH-Bell)



W — hermitian Operator

separable

$\mathrm{Tr}(\rho W) > 0$

W: entanglement witness

ρ is separable $\Rightarrow$ $\mathrm{Tr}(\rho W) \geq 0$

$\mathrm{Tr}(\rho W) < 0$ $\Rightarrow$ ρ is not separable

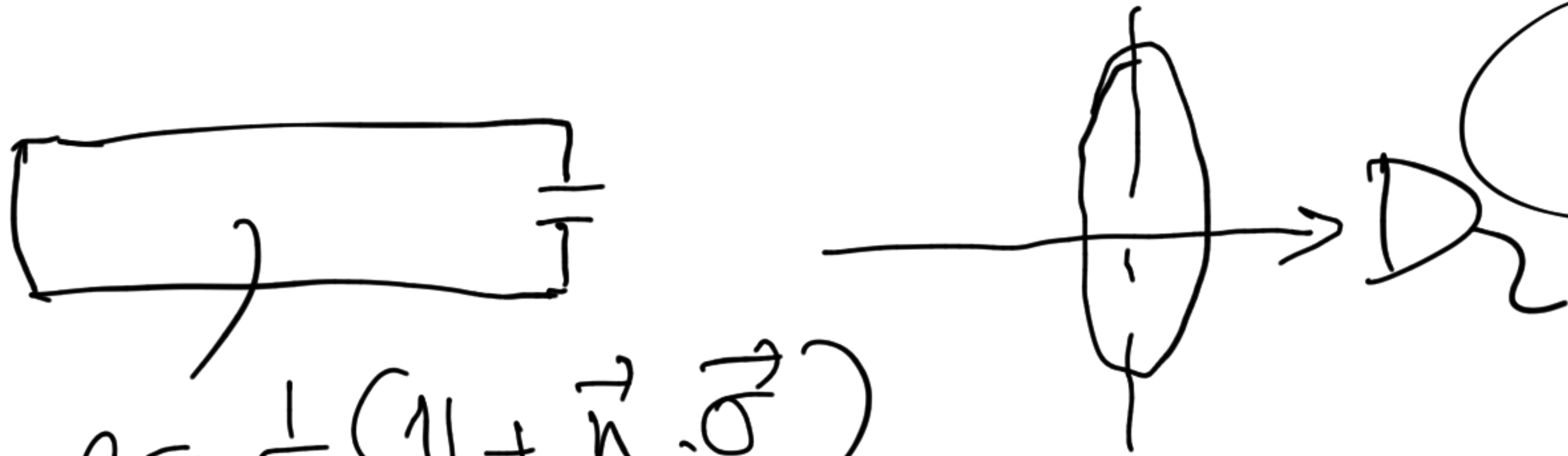
$$B = \left| E(\theta_1, \phi_1) + E(\theta_1, \phi_2) + E(\theta_2, \phi_1) - E(\theta_2, \phi_2) \right|$$

$$\underbrace{\mathrm{Tr}(\rho\, \vec{n}_1 \cdot \vec{\sigma} \otimes \vec{m}_1 \cdot \vec{\sigma})}_{\text{see Areeya's talk *}}$$

$$\left| \mathrm{Tr}\Big(\rho\, \vec{n}_1 \cdot \vec{\sigma} \otimes \vec{m}_1 \cdot \vec{\sigma} + \rho\, \vec{n}_1 \cdot \vec{\sigma} \otimes \vec{m}_2 \vec{\sigma} + \rho\, \vec{n}_2 \vec{\sigma} \otimes \vec{m}_1 \vec{\sigma} - \rho\, \vec{n}_2 \cdot \vec{\sigma} \otimes \vec{m}_2 \cdot \vec{\sigma}\Big) \right| \leq 2\,\mathrm{Tr}\rho$$

$\rho\, \hat{B}$

$$2\,\mathrm{Tr}\rho - \mathrm{Tr}(\rho \hat{B}) \geq 0 \qquad \mathrm{Tr}(\rho\, \underbrace{(2\mathbb{1} - \hat{B})}_{W}) \geq 0$$



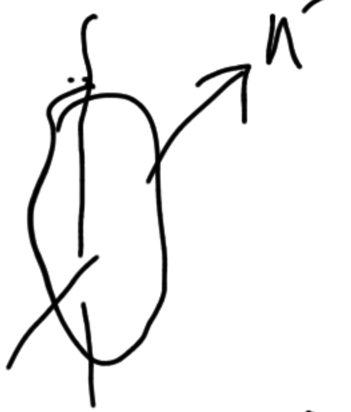
$$\underbrace{\rho = \frac{1}{2}\left(\mathbb{1} + \vec{n}\cdot\vec{\sigma}\right)}$$

$P(C)$ ^{1} or $P(nc) = \underbrace{1 - P(C)}_{-1}$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Tomography $\underline{n_x}, n_y, n_z$

$$\begin{array}{c|cc} & \langle V| & \langle H| \\ \hline |V\rangle & O & \\ |H\rangle & & \end{array}$$



$$Tr(\rho\sigma_z) = (1)P(C) + (-1)\,P(nc)$$

$$n_z = \underbrace{2P(C) - 1}$$

$$Tr(\rho\cdot\vec{n}\vec{\sigma})$$

$$n_x = Tr(\rho\sigma_x) \quad \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}} \; \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}}$$

$$\underbrace{n_y = Tr(\rho\sigma_y)}$$

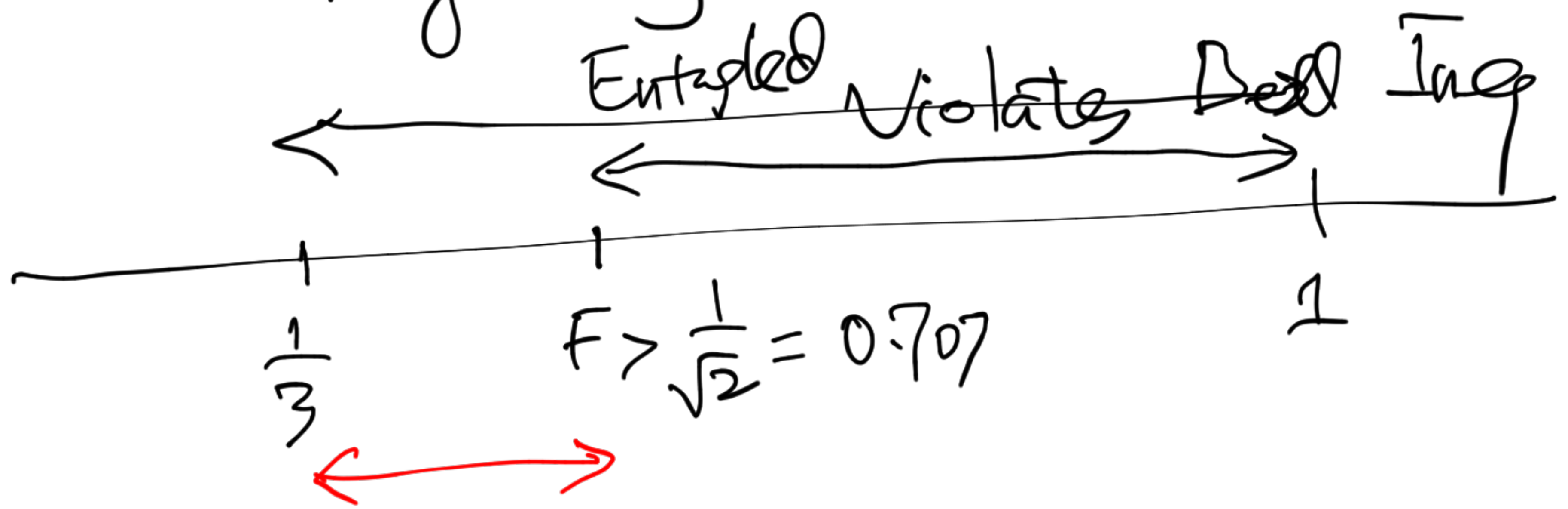
$$\rho_{2qubits} = \mathrm{Tr}\left(\rho\, \vec{n}\cdot\vec{\sigma} \otimes \vec{m}\cdot\vec{\sigma}\right)$$

$\vec{n}\cdot\vec{\sigma}$

Inequality: $\left| E(\theta_1, \phi_1) + E(\theta_1, \phi_2) + E(\theta_2, \phi_1) - E(\theta_2, \phi_2) \right| \leq 2$

Exercise: $\rho = F\,|\psi^-\rangle\langle\psi^-| + \frac{1-F}{4}\mathbb{1}$ Werner

Negativity or Concurrence



Note: $|E(\theta_1, \phi_1) + E(\theta_1, \phi_2) + E(\theta_2, \phi_1) - E(\theta_2, \phi_2)| \leq 2$

CHSH

probabilities

$p(c, c) + p(nc, nc) - p(c, nc) - p(nc, c)$

2 qubits

CHSH Correlation $\Longleftrightarrow$ CH

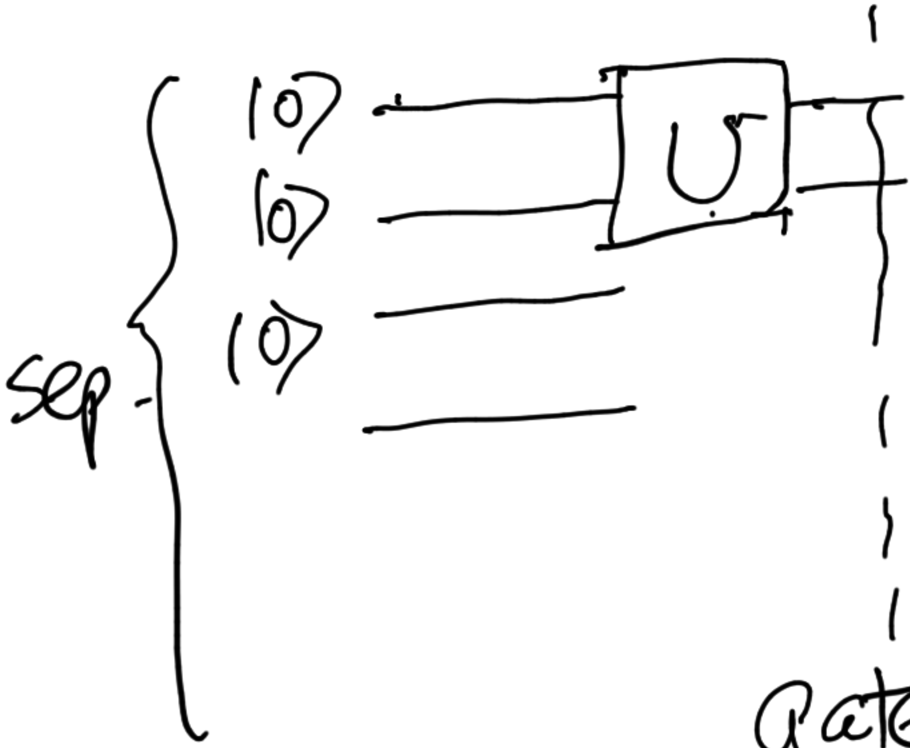
probabilities

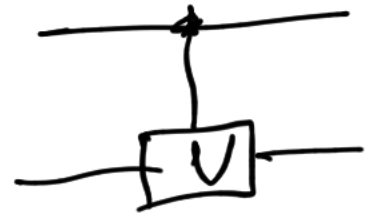
(Fine, 70s)

higher dimension $\underbrace{\text{Correlation-Type}}$ $\Longleftarrow$ Prob

	no of parties	dimA	dimB	Measurement Setting	Values	Type
CHSH	2	2	2	2	2	Corr.
CGLMP	2	d	d	2	2	Corr.
CH	2	2	2	2	2	Prob
Colin-Gisin-Linden-Massar-Popescu						
"Ours"	2	3	3	2	2	Prob
	N	2dim		2	2	

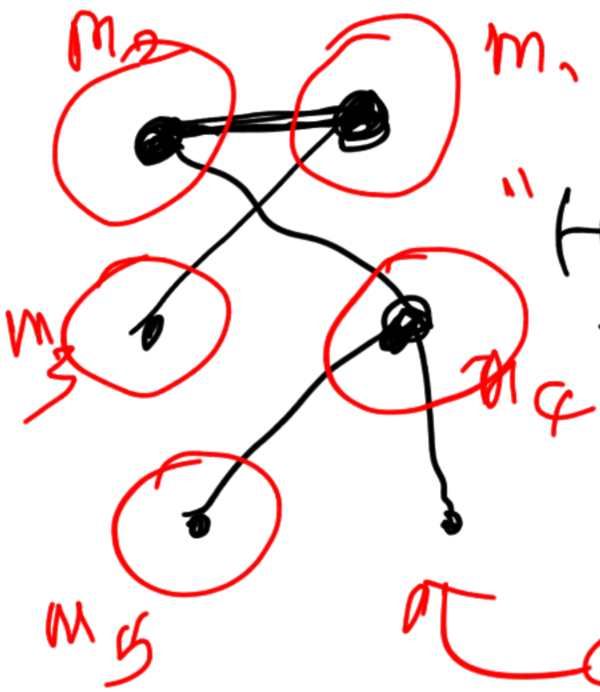
Zukowski-Brukner-Werner-Wolf
(WWZB)





entangling

gate (unitary) model



"Highly entangled"

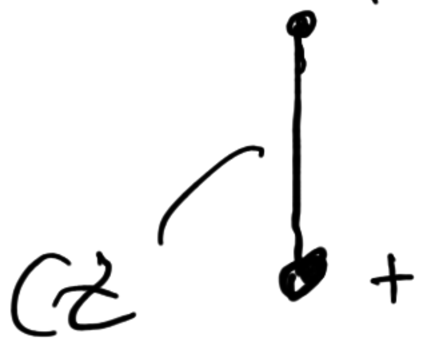
One-way computation

answer

One-way Computation:

Cluster state / graph state

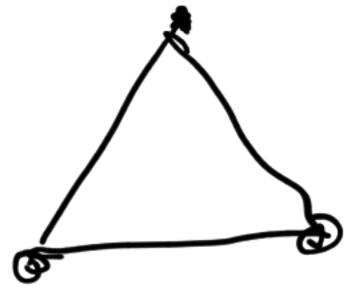
$$+ = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



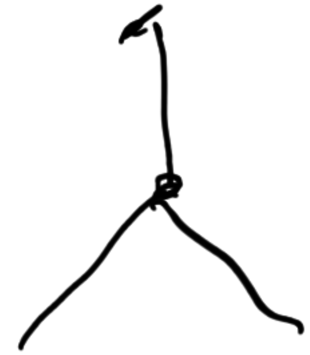
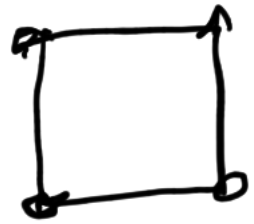
$$\left(\frac{1}{\sqrt{2}}\right)^2 (|0\rangle + \underline{|1\rangle})(|0\rangle + |1\rangle)$$

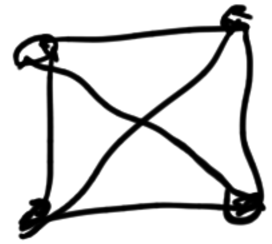
$$\xrightarrow[CZ]{} \left(\frac{1}{\sqrt{2}}\right)^2 (|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

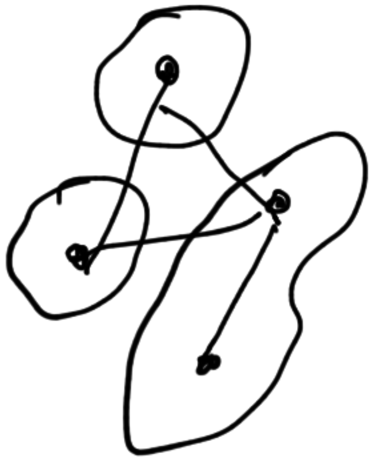
$$\left(\frac{1}{\sqrt{2}}\right)^2 \underbrace{(\underline{|0\rangle}(|0\rangle + |1\rangle) + \underline{|1\rangle}(|0\rangle - |1\rangle))}_{1}$$

 $\equiv$ 

(GHZ)

 $\neq$  

GHZ_4 



$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\phi_1\phi_2\phi_3\rangle + |\phi_1^\perp\phi_2^\perp\phi_3^\perp\rangle\right)$$

Raussendorf

Briegel

"Localized Entanglement"